Control and Coordination of Multi-Agent Systems

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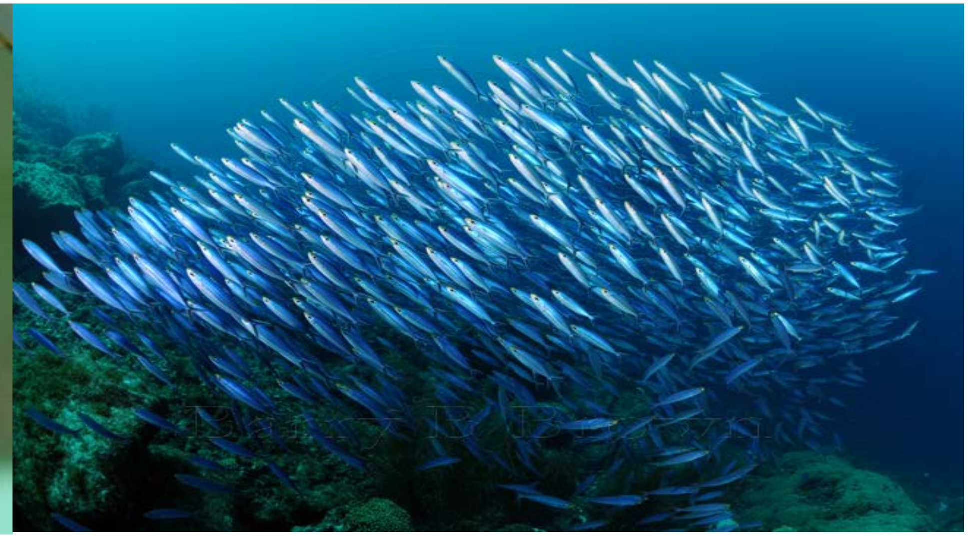
A (Swiss) Mood Picture

Courtesy of Alcherio Martinoli

Why Multi-Robot Systems?

- Strength in numbers
- Lots of (potential) applications
- Confluence of technology and algorithms
- ***Scientifically interesting!***

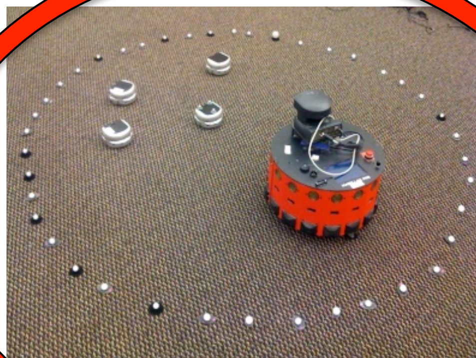
But How?

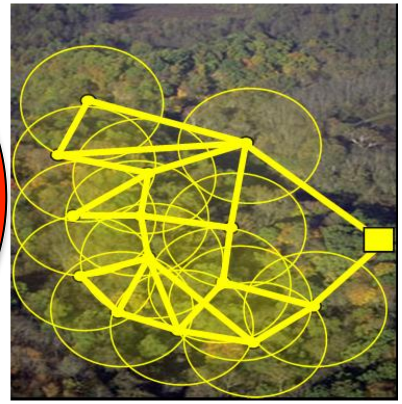
- Local (distributed)
- Scalable (decentralized)
- Safe and Reactive
- Emergent (but not too much)

Lynch, *Distributed Algorithms*, 1996.

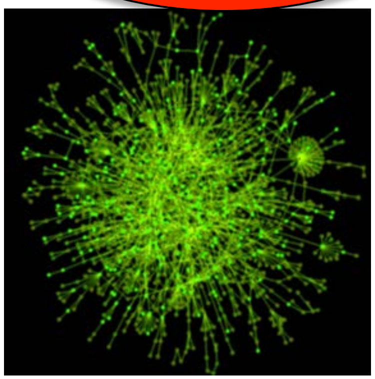
Application Domains



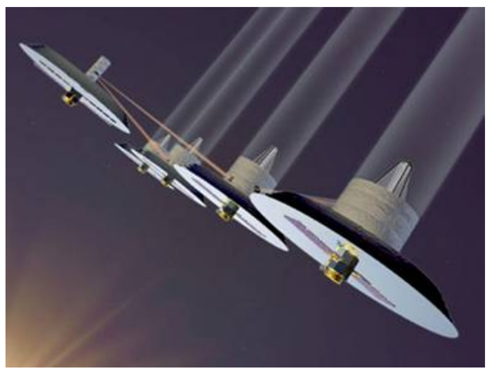
Multi-agent robotics



Sensor and communications networks

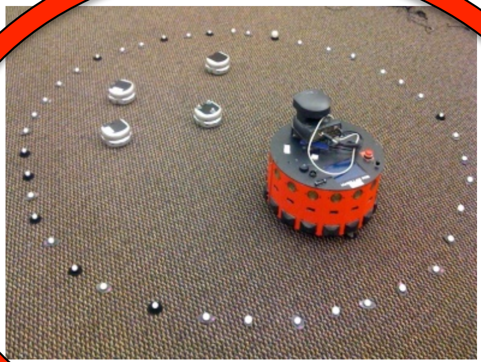


Biological networks



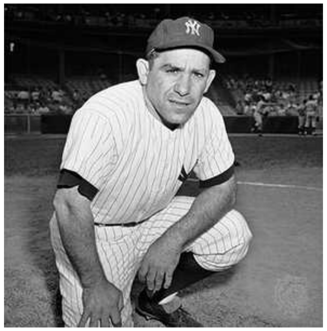
Coordinated control

Application Domains

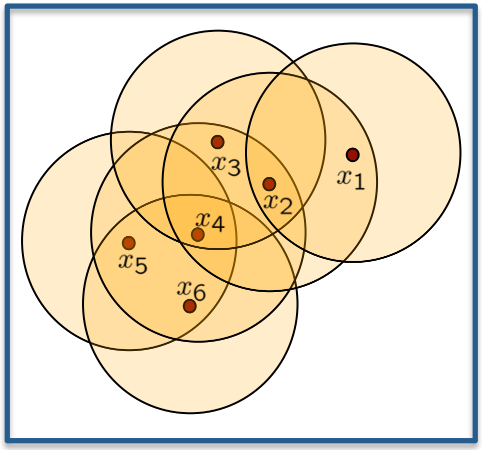


Multi-agent robotics

"There is nothing more practical than a good theory" - James C. Maxwell (Lewin? Pauling?)

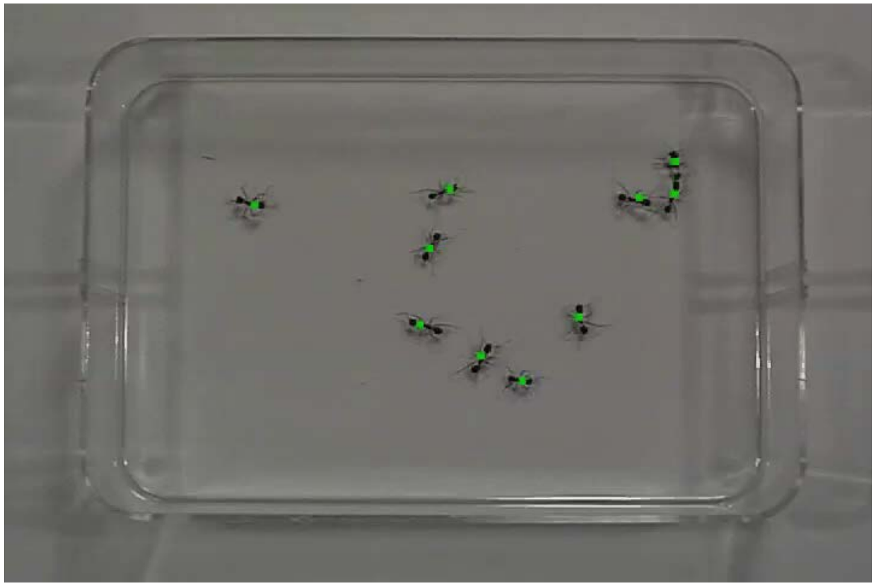


"In theory, theory and practice are the same.
In practice, they are not" – Yogi Berra

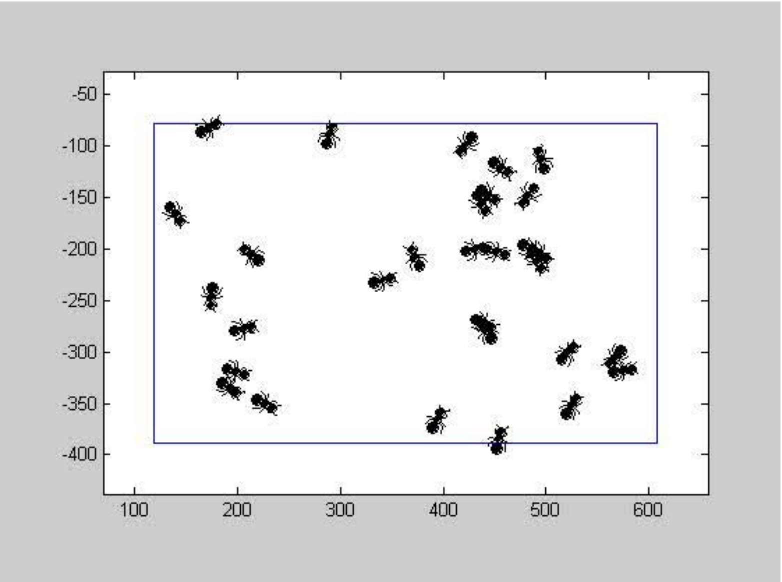


1. **GRAPH-BASED ABSTRACTIONS**
2. FORMATION CONTROL
3. INTERACTING WITH NETWORKS

A True Swarm



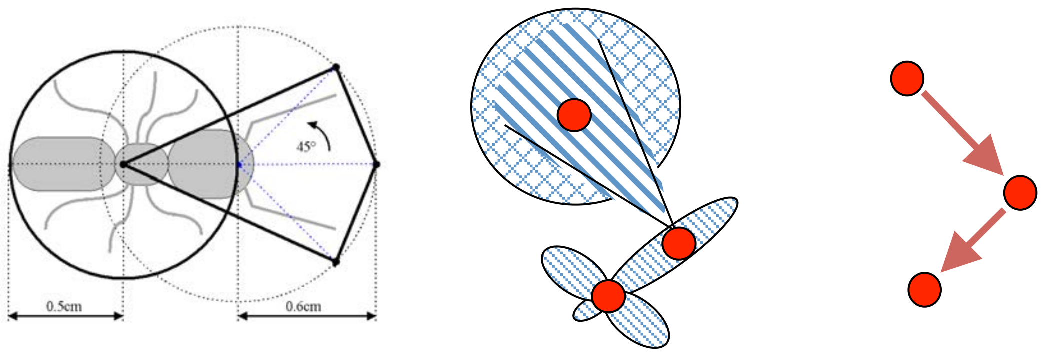
?



"They look like ants."
– Stephen Pratt, Arizona State University

Graphs as Network Abstractions

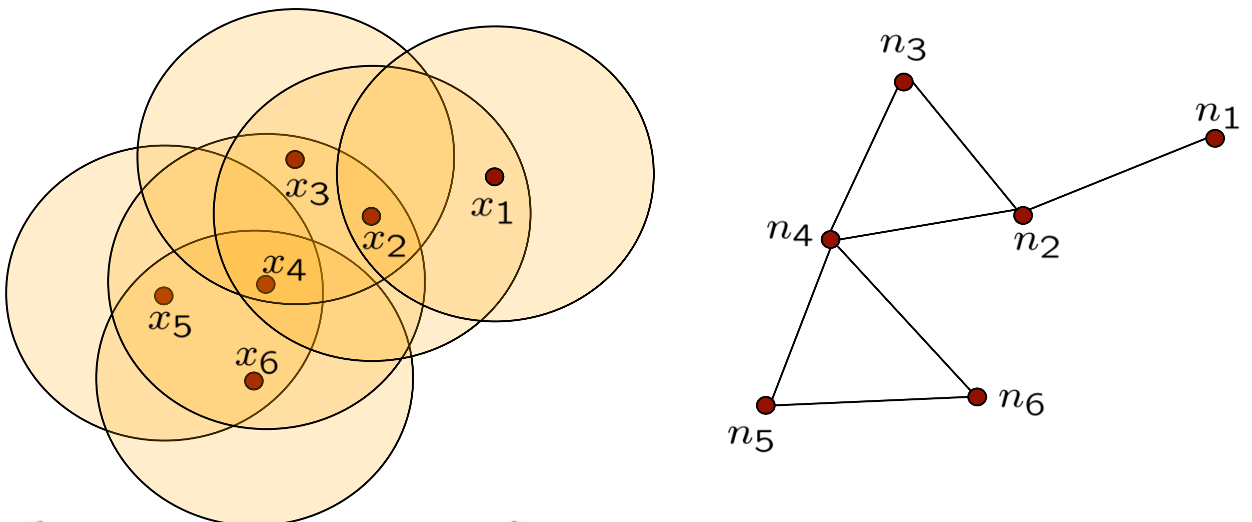
- A networked sensing and actuation system consists of
  - **NODES** - physical entities with limited resources (computation, communication, perception, control)
  - **EDGES** - virtual entities that encode the flow of information between the nodes



- The "right" mathematical object for characterizing such systems at the network-level is a **GRAPH**
  - Purely ***combinatorial*** object (no ***geometry*** or ***dynamics***)
  - The characteristics of the information flow is abstracted away through the (possibly weighted and directed) edges

Graphs as Network Abstractions

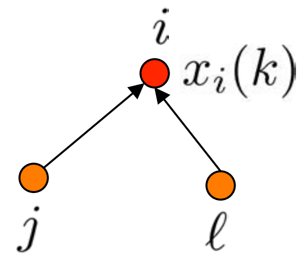
- The connection between the combinatorial graphs and the geometry of the system can for instance be made through geometrically defined edges.
- Examples of such proximity graphs include **disk-graphs**, **Delaunay graphs, visibility graphs**, and **Gabriel graphs**[1].



$$\mathcal{N} = \{n_1, n_2, n_3, n_4, n_5, n_6\}$$
$$\mathcal{E} = \{(n_1, n_2), (n_2, n_3), (n_3, n_4), (n_2, n_4), (n_4, n_5), (n_4, n_6), (n_5, n_6)\}$$

The Basic Setup

- $x_i(k)$ = "state" at node i at time k
- $N_i(k)$ = "neighbors" to agent i



- Information "available to agent i

$$I_i^c(k) = \{x_j(k) \mid j \in N_i(k)\} \longleftarrow \text{common ref. frame (comms.)}$$

or

$$I_i^r(k) = \{x_i(k) - x_j(k) \mid j \in N_i(k)\} \longleftarrow \text{relative info. (sensing)}$$

- Update rule:

$$x_i(k+1) = F_i(x_i(k), I_i(k)) \longleftarrow \text{discrete time}$$

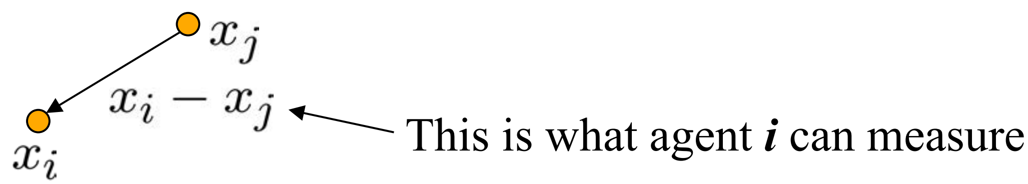
or

$$\dot{x}_i(t) = F_i(x_i(t), I_i(t)) \longleftarrow \text{continuous time}$$

- ***How pick the update rule?***

Rendezvous – A Canonical Problem

- Given a collection of mobile agents who can only measure the relative displacement of their neighbors (no global coordinates)



- Problem: Have all the agents meet at the same (unspecified) position

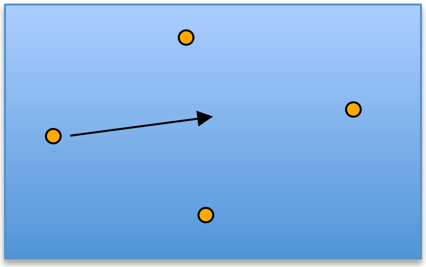
- If there are only two agents, it makes sense to have them drive towards each other, i.e.

$$\dot{x}_1 = -\gamma_1(x_1 - x_2)$$
$$\dot{x}_2 = -\gamma_2(x_2 - x_1)$$

- If $\gamma_1 = \gamma_2$ they should meet halfway

Rendezvous – A Canonical Problem

- If there are more than two agents, they should probably aim towards the centroid of their neighbors (or something similar)



$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j)$$

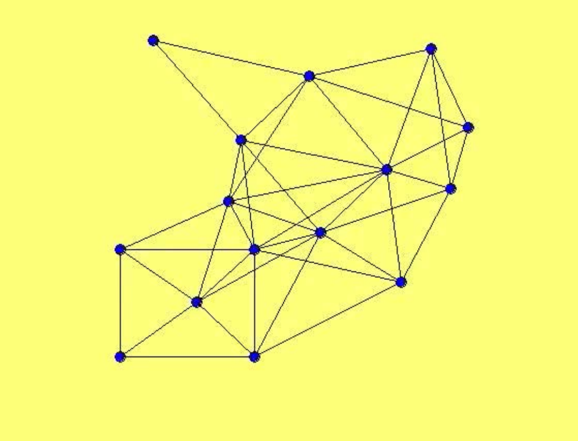
The "consensus protocol" drives all states to the same value if the interaction topology is "rich enough"

Tsitsiklis 1988, Bertsekas, Tsitsiklis, 1989. Jadbabaie, Lin, Morse, 2003. Olfati-Saber, Murray, 2003.

Rendezvous – A Canonical Problem

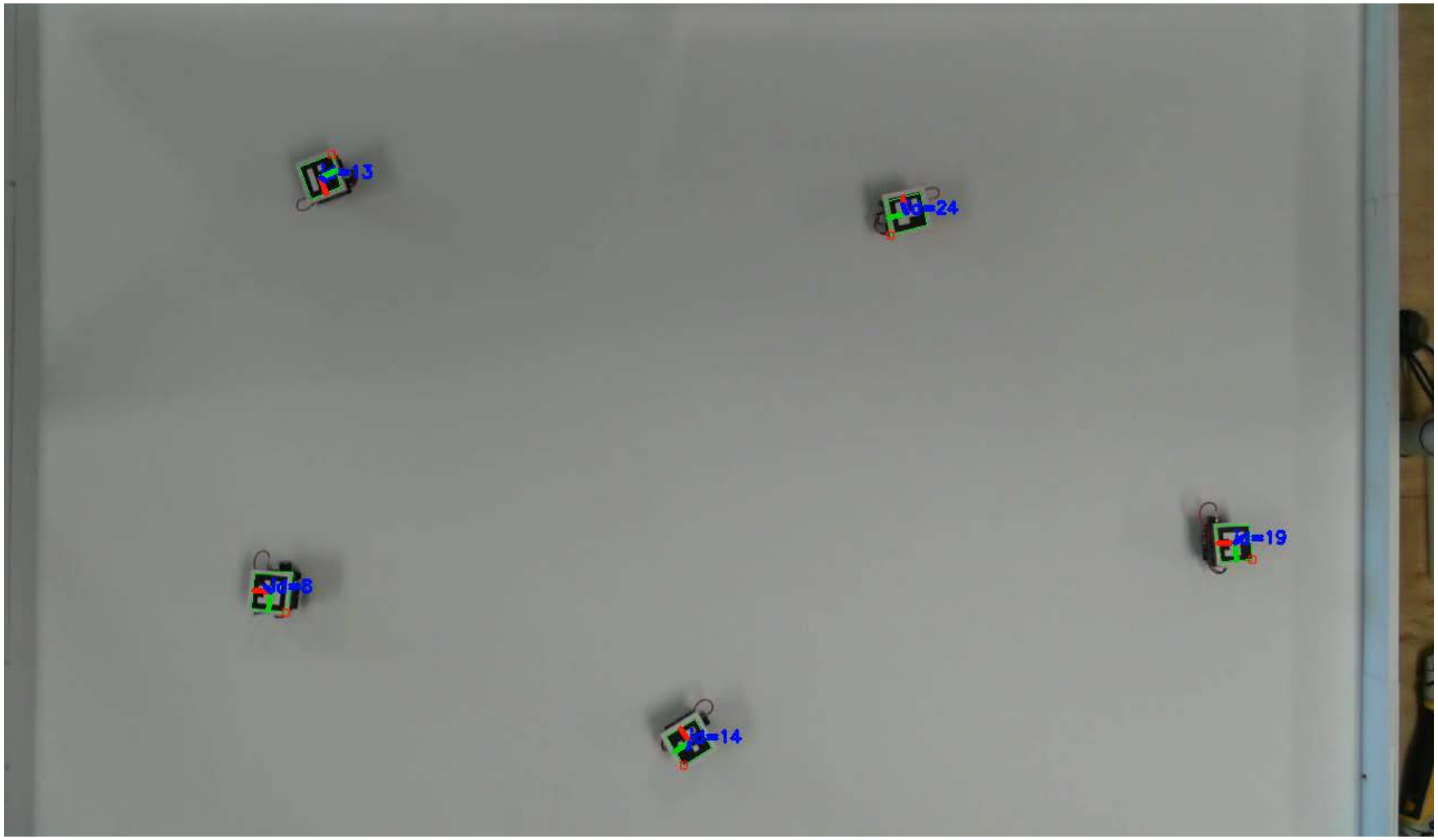
Fact [2-4]: If and only if the graph* is connected, the ***consensus equation*** drives all agents to the same state value

$$\lim_{t\to\infty} x_i(t) = \bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j(0)$$



*static and undirected graphs

Consensus/Rendezvous



Pickem, Squires, Egerstedt, 2015

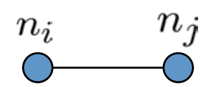
Algebraic Graph Theory

- To show this, we need some tools…
- ***Algebraic graph theory*** provides a bridge between the combinatorial graph objects and their matrix representations
  - **Degree matrix**:

$$D = \mathrm{diag}(\deg(n_1), \ldots, \deg(n_N))$$

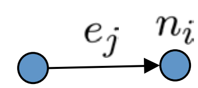
  - **Adjacency matrix**:

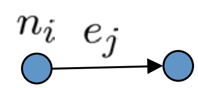
$$A = [a_{ij}], \;\; a_{ij} = \begin{cases} 1 & \text{if} \\ 0 & \text{o.w.} \end{cases}$$



  - **Incidence matrix** (directed graphs):

$$\mathcal{I} = [\iota_{ij}], \;\; \iota_{ij} = \begin{cases} 1 & \text{if} \\ -1 & \text{if} \\ 0 & \text{o.w.} \end{cases}$$





  - ***Graph Laplacian***:

$$\mathcal{L} = D - A = \mathcal{I}\mathcal{I}^T$$

The Consensus Equation

- One reason why the graph Laplacian is so important is through the already seen "consensus equation"

$$\dot{x}_i = -\sum_{j \in \mathcal{N}_i} (x_i - x_j), \; i = 1, \ldots, N$$

or equivalently (W.L.O.G. scalar agents)

$$\left.\begin{array}{l} \dot{x}_i = -\mathrm{deg}(n_i) x_i + \sum_{j=1}^{N} a_{ij} x_j \\ x = \left[\begin{array}{cccc} x_1 & x_2 & \cdots & x_N \end{array}\right]^T \end{array}\right\} \Rightarrow \; \dot{x} = -\mathcal{L} x$$

- This is an autonomous LTI system whose stability properties depend purely on the spectral properties of the Laplacian.

Graph Laplacians: Useful Properties

- It is orientation independent
- It is symmetric and positive semi-definite
- If the graph is ***connected*** then

$$\text{eig}(\mathcal{L}) = \{\lambda_1, \ldots, \lambda_N\}, \text{ with } 0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$$

$$\text{eigv}(\mathcal{L}) = \{\nu_1, \ldots, \nu_N\}, \text{ with } null(\mathcal{L}) = \text{span}\{\nu_1\} = \text{span}\{\mathbf{1}\}$$

$$\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

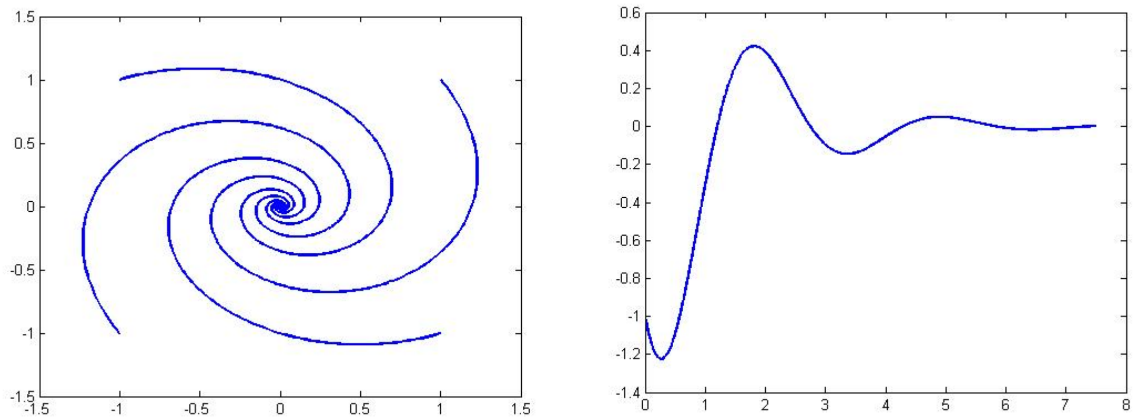
Stability: Basics

- The stability properties (what happens as time goes to infinity?) of a linear, time-invariant system is completely determined by the eigenvalues of the system matrix

$$\dot{x} = Ax \qquad (\dot{x} = -Lx)$$

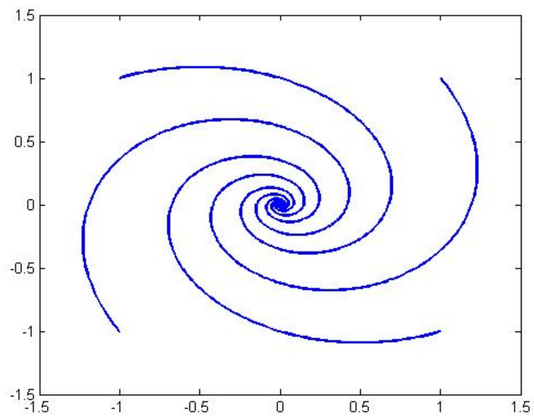
- Eigenvalues $\lambda(A) = \lambda_1, \ldots, \lambda_n$

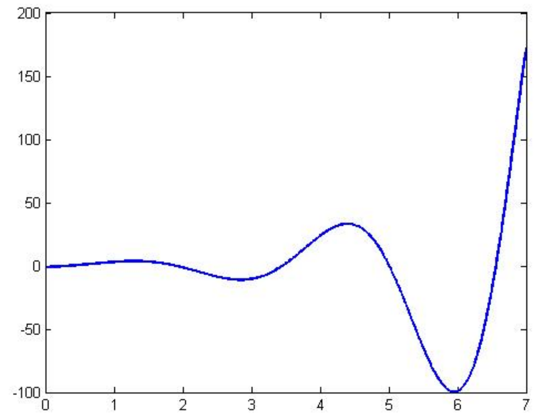
- Asymptotic stability: $\mathrm{Re}(\lambda_i) < 0, \; i = 1, \ldots, n \; \Rightarrow \; \lim_{t \to \infty} x(t) = 0$



Stability: Basics

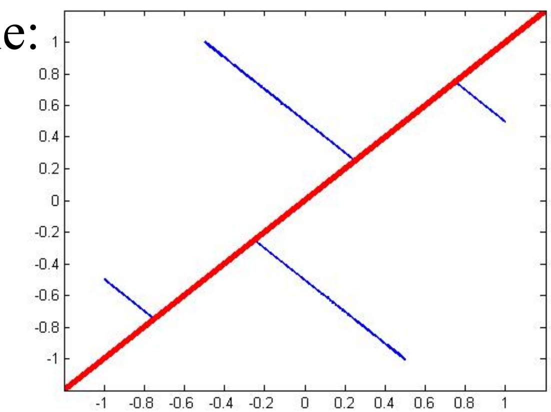
- Unstable: $\exists i$ s.t. $\mathrm{Re}(\lambda_i) > 0 \;\Rightarrow\; \exists x(0)$ s.t. $\lim_{t\to\infty} \|x(t)\| = \infty$





- (A special case of) Critically stable:

$$0 = \lambda_1 > \lambda_2 \geq \ldots \geq \lambda_n, \;\Rightarrow$$
$$\lim_{t\to\infty} x(t) \in \text{ null}(A)$$



This is the case for the consensus equation

Static and Undirected Consensus

- If the graph is static and connected, under the consensus equation, the states will reach *null(L)*
- Fact (again):

$$\mathrm{null}(L) = \mathrm{span}\{\mathbf{1}\}, \;\; x \in \mathrm{null}(L) \;\Leftrightarrow\; x = \begin{bmatrix} \alpha \\ \alpha \\ \vdots \\ \alpha \end{bmatrix}, \;\alpha \in \Re$$

- So all the agents state values will end up at the same value, i.e. the consensus/rendezvous problem is solved!

$$\dot{x}_i = -\sum_{j \in N_i} (x_i - x_j) \;\Rightarrow\; \lim_{t \to \infty} x_i(t) = \frac{1}{n} \sum_{j=1}^{n} x_j(0) = \frac{1}{n} \mathbf{1}^T x(0)$$

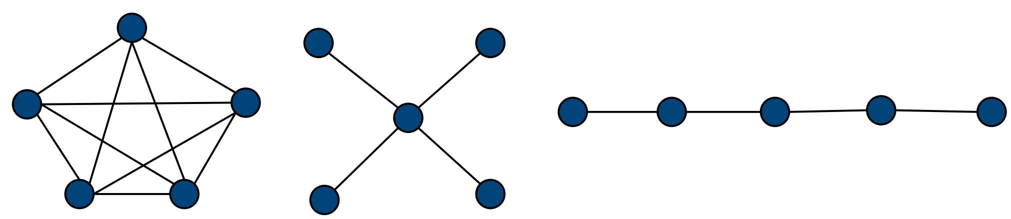
Convergence Rates

- The second smallest eigenvalue of the graph Laplacian is really important!
- Algebraic Connectivity (= 0 if and only if graph is disconnected)
- Fiedler Value (robustness measure)
- ***Convergence Rate:***

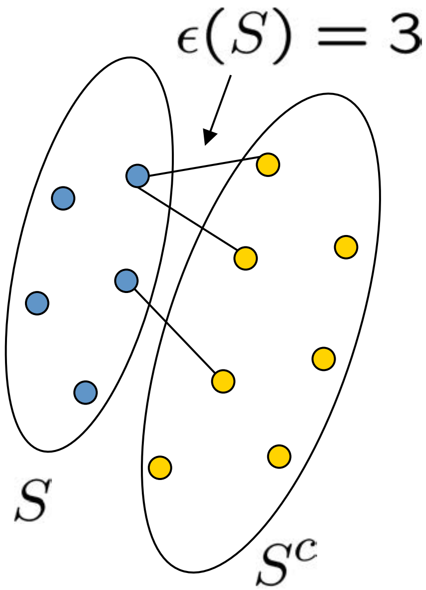
$$\|x(t) - \frac{1}{n}\mathbf{1}\mathbf{1}^T x(0)\| \leq Ce^{-\lambda_2 t}$$

- **Punch-line:** The more connected the network is, the faster it converges (and the more information needs to be shuffled through the network)

- Complete graph: $\lambda_2 = n$
- Star graph: $\lambda_2 = 1$
- Path graph: $\lambda_2 < 1$



Cheeger's Inequality



$$\phi(S) = \frac{\epsilon(S)}{\min\{|S|, |S^c|\}}$$

(measures how many edges need to be cut to make the two subsets disconnected as compared to the number of nodes that are lost)

isoperimetric number:

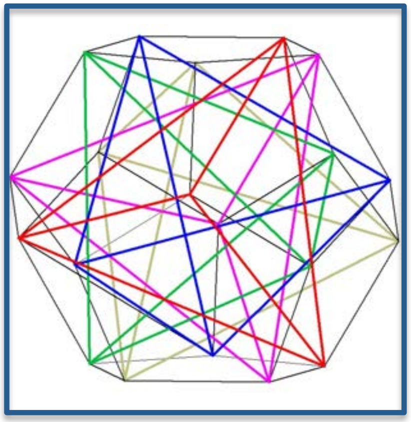
$$\phi(G) = \min_S \phi(S)$$

(measures the robustness of the graph)

$$\phi(G) \geq \lambda_2 \geq \frac{\phi(G)^2}{2\Delta(G)}$$

Summary I

- Graphs are natural abstractions (combinatorics instead of geometry)
- Consensus problem (and equation)
- Static Graphs:
  - Undirected: Average consensus iff G is connected
- ***Need richer network models and more interesting tasks!***



1. GRAPH-BASED ABSTRACTIONS
2. FORMATION CONTROL
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Formation Control v.1

- Being able to reach consensus goes beyond solving the rendezvous problem.
- Formation control:

$$\underset{\text{agent positions}}{x_1, \ldots, x_N} \longrightarrow \underset{\text{target positions}}{y_1, \ldots, y_N}$$

- But, formation achieved if the agents are in any translated version of the targets, i.e.,

$$x_i = y_i + \tau, \ \forall i, \ \text{for some } \tau$$

- Enter the consensus equation [5]:

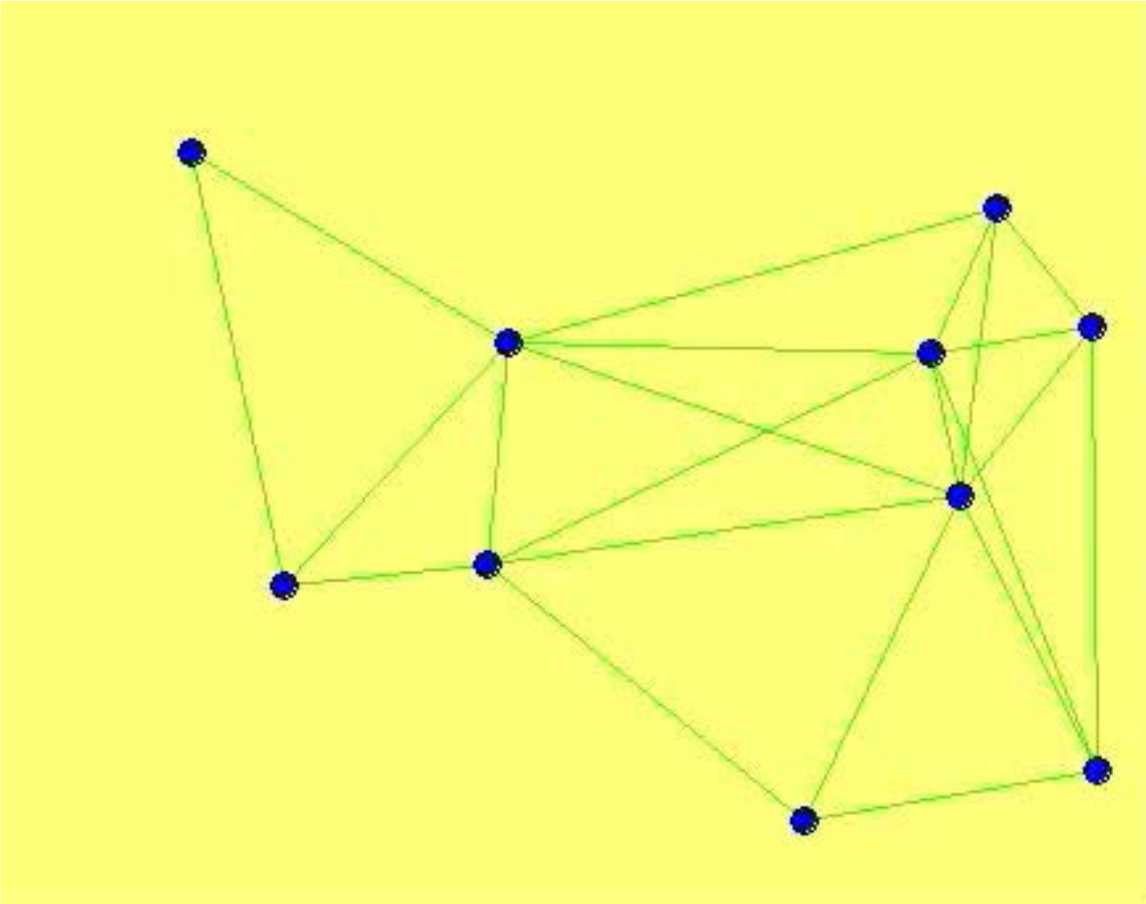
$$e_i = x_i - y_i$$

$$\dot{e}_i = -\sum_{j \in N_i} (e_i - e_j)$$

$$e_i(\infty) = e_j(\infty) = \tau$$

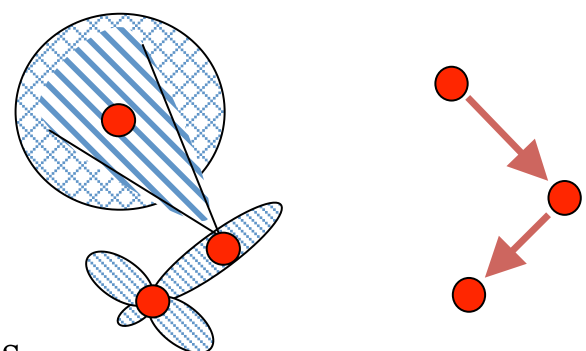
$$\boxed{\begin{aligned} \dot{x}_i &= \sum_{j \in N_i} [(x_i - x_j) - (y_i - y_j)] \\ x_i(\infty) &= y_i + \tau, \ \forall i \end{aligned}}$$

Formation Control v.1



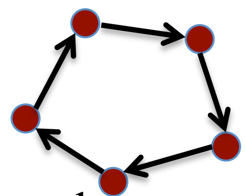
Beyond Static and Undirected Consensus

- So far, the consensus equation will drive the node states to the same value if the graph is static and connected.
- But, this is clearly not the case for mobile agents in general:
  - **Edges = communication links**
    - Random failures
    - Dependence on the position (shadowing,…)
    - Interference
    - Bandwidth issues
  - **Edges = sensing**
    - Range-limited sensors
    - Occlusions
    - Weirdly shaped sensing regions

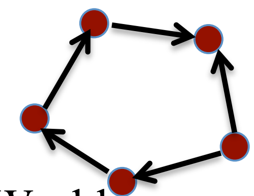


Directed Graphs

- Instead of connectivity, we need directed notions:
  - ***Strong connectivity*** = there exists a directed path between any two nodes
  - ***Weak connectivity*** = the disoriented graph is connected



Strongly connected



Weakly connected

- Directed consensus:

$$\dot{x}_i = -\sum_{j \in N_i^{in}} (x_i - x_j)$$

Directed Consensus

- Undirected case: Graph is connected = sufficient information is flowing through the network
- Clearly, in the directed case, if the graph is strongly connected, we have the same result

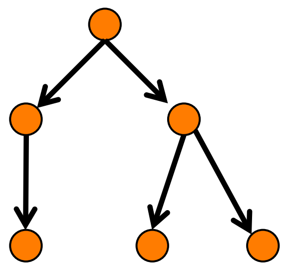
- Theorem: If G is strongly connected, the consensus equation achieves

$$\lim_{t \to \infty} (x_i - x_j) = 0, \ \forall i, j$$

- This is an unnecessarily strong condition! Unfortunately, weak connectivity is too weak.

Spanning, Outbranching Trees

- Consider the following structure



- Seems like all agents should end up at the root node
- Theorem [2]: Consensus in a static and directed network is achieved if and only if G contains a ***spanning, outbranching tree***.

Where Do the Agents End Up?

- Recall: Undirected case

$$\lim_{t\to\infty} x_i(t) = \bar{x}(0) = \frac{1}{N}\sum_{j=1}^{N} x_j(0), \ \forall i$$

- How show that?
- The centroid is invariant under the consensus equation

$$\dot{\bar{x}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\in N_i}(x_j - x_i) = 0$$

- And since the agents end up at the same location, they must end up at the static centroid (average consensus).

Where Do the Agents End Up?

- When is the centroid invariant in the directed case?

$$q^T L = 0, \; w = q^T x \;\Rightarrow\; \dot{w} = q^T \dot{x} = -q^T L x = 0$$

- w is invariant under the consensus equation
- The centroid is given by

$$\bar{x} = \frac{1}{N}\mathbf{1}^T x$$

which thus is invariant if

$$\mathbf{1}^T L = 0$$

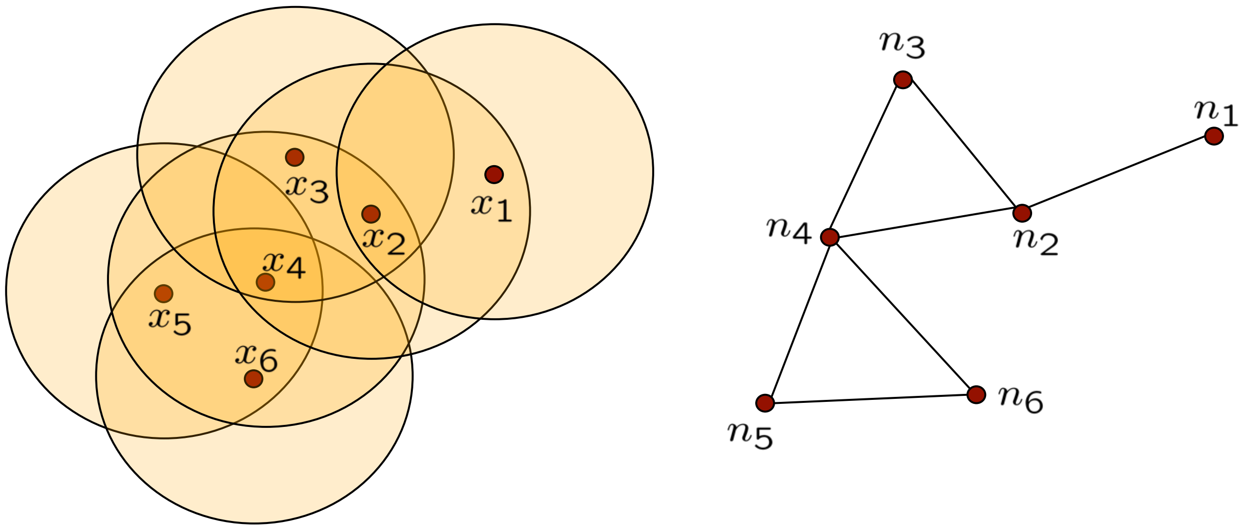
- Def: G is balanced if

$$deg^{in}(i) = deg^{out}(i), \; \forall i \in V \;\Leftrightarrow\; \mathbf{1}^T L = 0$$

- **Theorem** [2]: If G is balanced and consensus is achieved then average consensus is achieved!

Dynamic Graphs

- In most cases, edges correspond to available sensor or communication data, i.e., the edge set is time varying



- We now have a switched system and spectral properties do not help for establishing stability
- Need to use Lyapunov functions

Lyapunov Functions

- Given a nonlinear system

$$\dot{x} = f(x)$$

- V is a (weak) Lyapunov function if

$$\begin{array}{ll} (i) & V(x) > 0, \;\; \forall x \neq 0 \\ (ii) & \dot{V}(x) = \dfrac{\partial V}{\partial x} f(x) < 0, \;\; \forall x \neq 0 \quad (\leq 0) \end{array}$$

- The system is asymptotically stable if and only if there exists a Lyapunov function
- [LaSalle's Invariance Principle] If it has a weak Lyapunov function the system converges asymptotically to the largest set with f=0 s.t. the derivative of V is 0

Switched Systems

- Similarly, consider a switched system

$$\dot{x} = f_\sigma(x), \quad \sigma(t) \in \{1, \ldots, q\}$$

- The system is *universally asymptotically stable* if it is asymptotically stable for all switch sequences

- A function V is a common Lyapunov function if it is a Lyapunov function to all subsystems

$$V > 0, \ \frac{\partial V}{\partial x} f_i < 0, \ i = 1, \ldots, q$$

- Theorem [9]: Universal stability if and only if there exists a common Lyapunov function. (Similar for LaSalle.)

Switched Networked Systems

- Switched consensus equation

$$\dot{x} = -L_\sigma x$$

- Consider the following candidate Lyapunov function

$$V(x) = \frac{1}{2}x^T x, \quad \dot{V}(x) = x^T \dot{x} = -x^T L_\sigma x$$

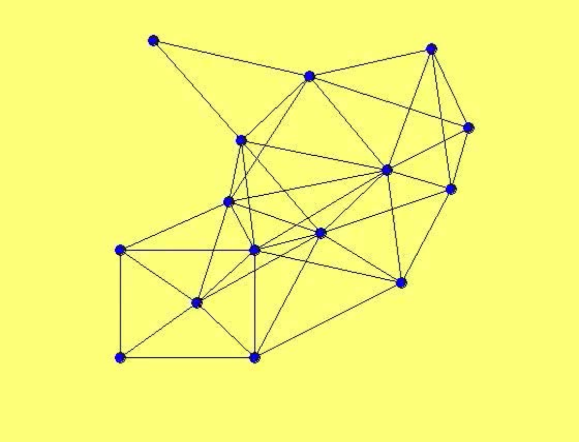
- This is a common (weak) Lyapunov function as long as *G* is connected for all times

- Using LaSalle's theorem, we know that in this case, it ends up in the null-space of the Laplacians

Switched (Undirected) Consensus

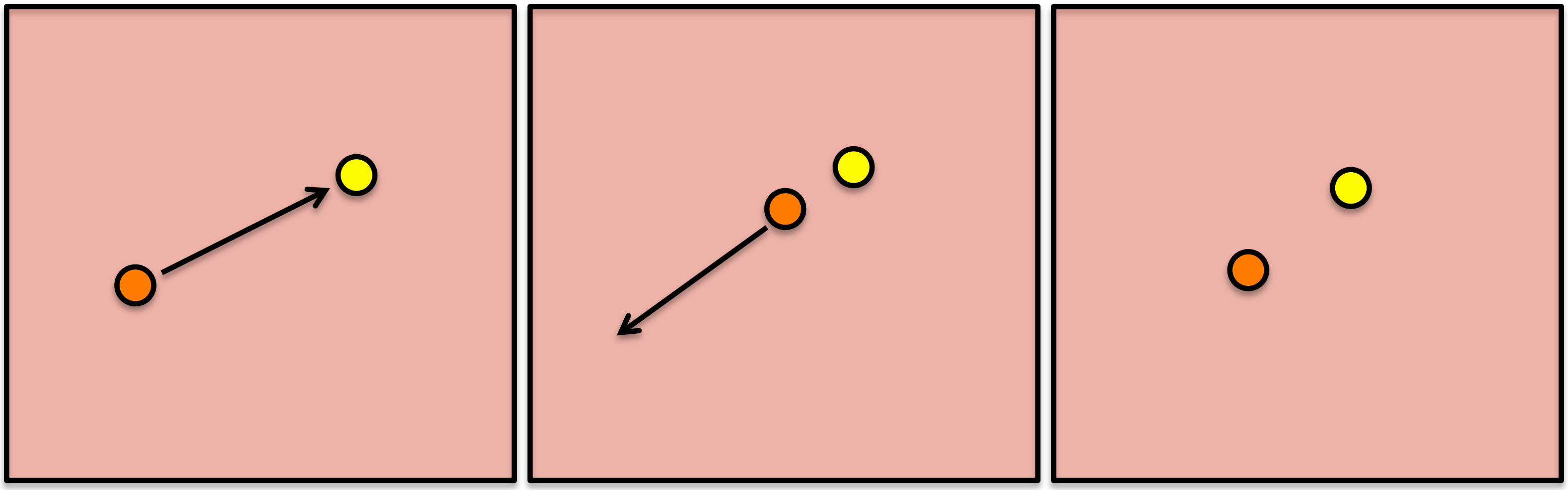
Theorem [2-4]: As long as the graph stays connected, the ***consensus equation*** drives all agents to the same state value

$$\lim_{t\to\infty} x_i(t) = \bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j(0)$$



Collisions?

Adding Weights



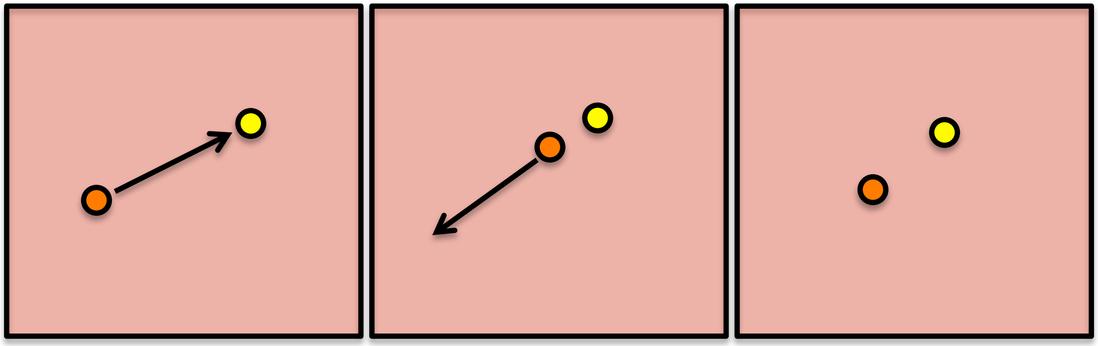
too far away

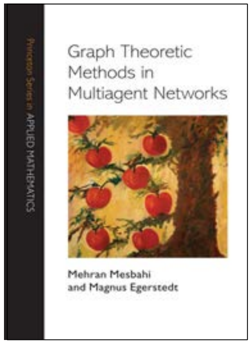
too close

just right

Adding Weights

Selecting the Weights





Mesbahi, Egerstedt 2010

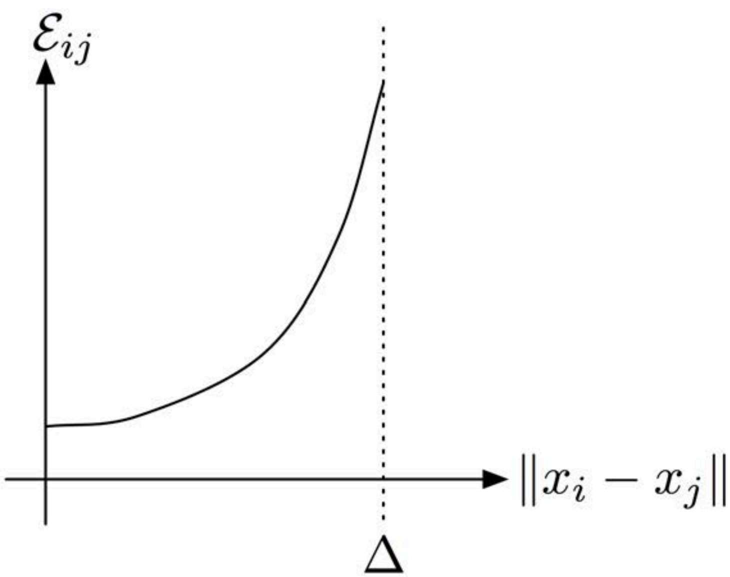
$$\dot{x}_i = -\sum_{j \in N_i} w_{i,j}(\|x_i - x_j\|)(x_i - x_j)$$

- Formation Control
- Connectivity Maintenance
- Coverage Control
- Flocking and Swarming
- Patrolling
- Pursuit/Evasion

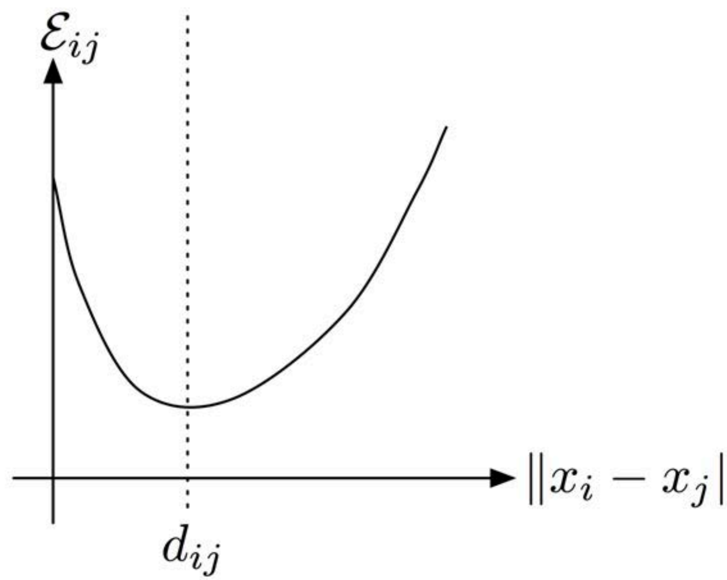
Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Weights Through Edge-Tensions

- How select appropriate weights?
- Let an edge tension be given by $\mathcal{E} = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{i,j} \mathcal{E}_{i,j}(\|x_i - x_j\|)$



Connectivity Maintenance



Formation Control

Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Weights Through Edge-Tensions

- How select appropriate weights?
- Let an edge tension be given by $\mathcal{E} = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{i,j}\mathcal{E}_{i,j}(\|x_i - x_j\|)$
- We get

$$\frac{\partial \mathcal{E}_{i,j}}{\partial x_i} = w_{i,j}(\|x_i - x_j\|)(x_i - x_j)$$

- Gradient descent

$$\dot{x}_i = -\frac{\partial \mathcal{E}}{\partial x_i} = -\sum_{j \in N_i} w_{i,j}(\|x_i - x_j\|)(x_i - x_j)$$

$$\frac{d\mathcal{E}}{dt} = \frac{\partial \mathcal{E}}{\partial x}\dot{x} = -\left\|\frac{\partial \mathcal{E}}{\partial x}\right\|^2$$

Energy is non-increasing!
(weak Lyapunov function)

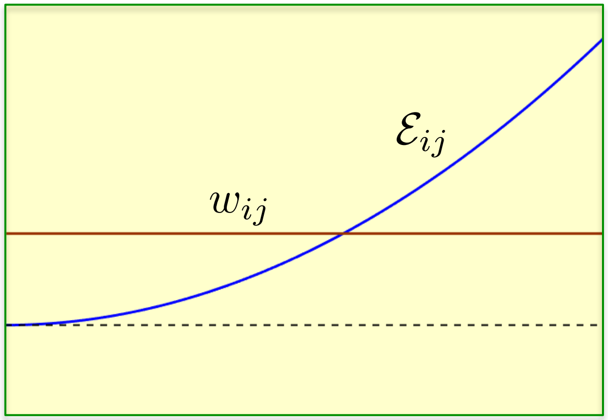
Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Examples

Standard, linear consensus!

$$\mathcal{E}_{ij} = \frac{1}{2}\|x_i - x_j\|^2 \;\Rightarrow\; w_{ij} = 1$$

$$\dot{x}_i = -\sum_{j \in N_i} (x_i - x_j)$$



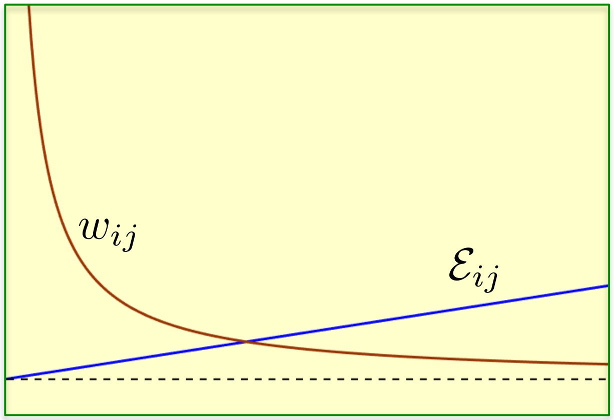
Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Examples

Unit vector (biology)

$$\mathcal{E}_{ij} = \|x_i - x_j\| \;\Rightarrow\; w_{ij} = \frac{1}{\|x_i - x_j\|}$$

$$\dot{x}_i = -\sum_{j \in N_i} \frac{x_i - x_j}{\|x_i - x_j\|}$$



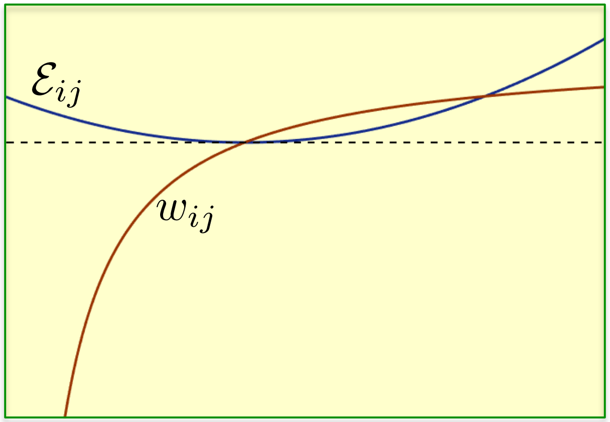
Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Examples

Formation control v.2

$$\mathcal{E}_{ij} = \frac{1}{2}(\|x_i - x_j\| - d_{ij})^2 \;\Rightarrow\; w_{ij} = \frac{\|x_i - x_j\| - d_{ij}}{\|x_i - x_j\|}$$

$$\dot{x}_i = -\sum_{j \in N_i} \frac{(\|x_i - x_j\| - d_{ij})(x_i - x_j)}{\|x_i - x_j\|}$$



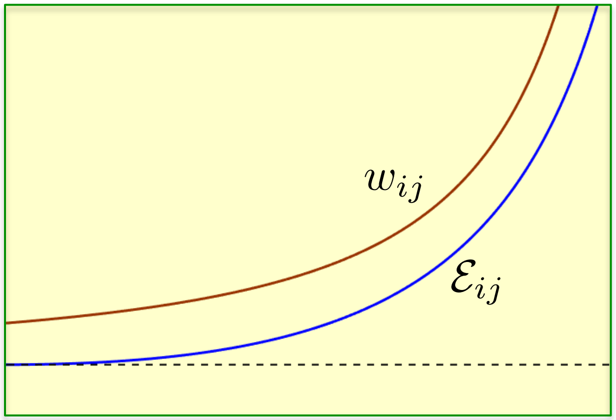
Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Examples

Connectivity maintenance

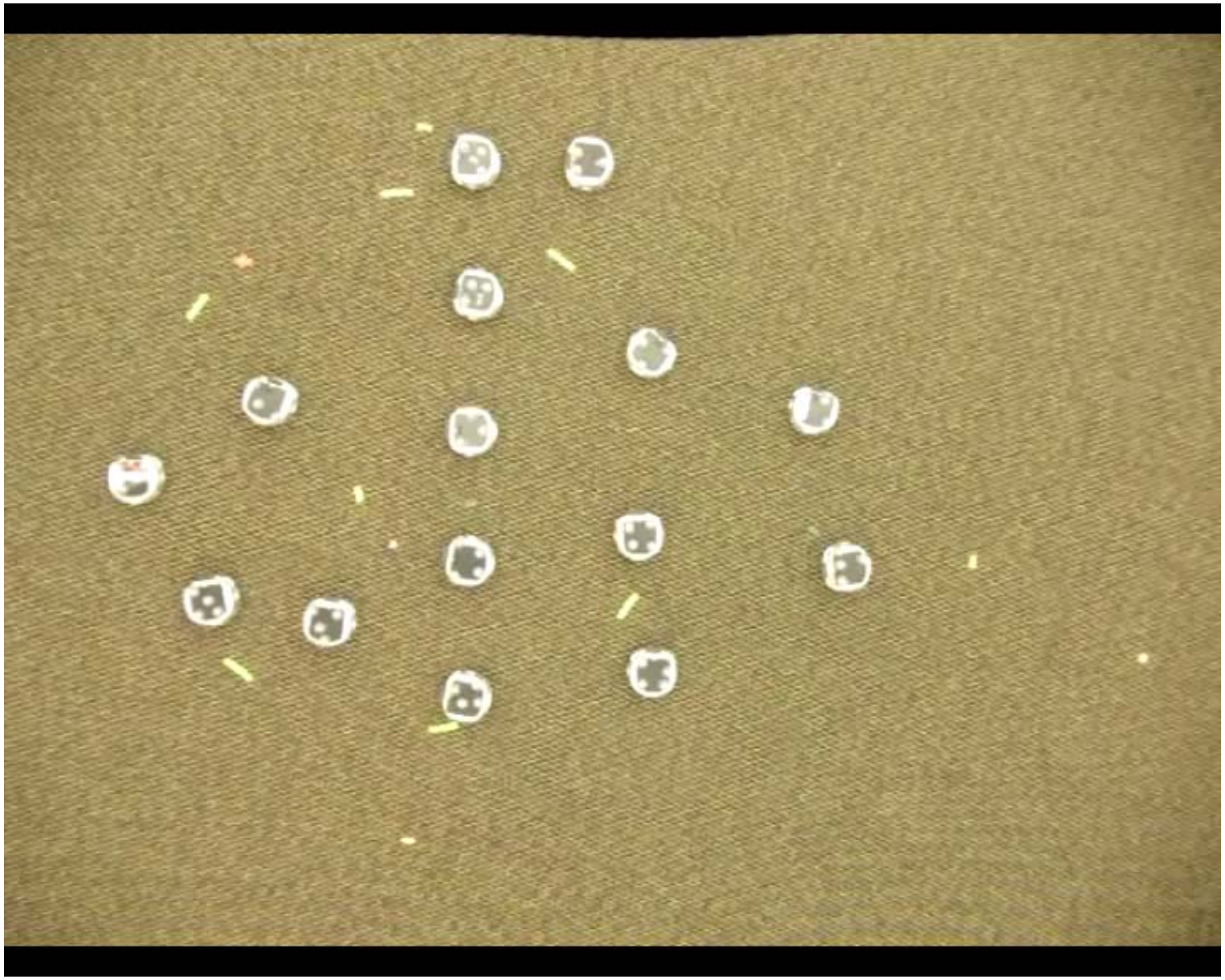
$$\mathcal{E}_{ij} = \frac{\|x_i - x_j\|^2}{\Delta - \|x_i - x_j\|} \Rightarrow w_{ij} = \frac{2\Delta - \|x_i - x_j\|}{(\Delta - \|x_i - x_j\|)^2}$$

$$\dot{x}_i = -\sum_{j \in N_i} \frac{(2\Delta - \|x_i - x_j\|)(x_i - x_j)}{(\Delta - \|x_i - x_j\|)^2}$$



Mesbahi, Egerstedt 2010. Guttal, Couzin 2011. Ji, Egerstedt, 2007. Bishop, Deghat, Anderson 2014. Zavlanos, Pappas 2008.

Weighted Consensus: Formation Control

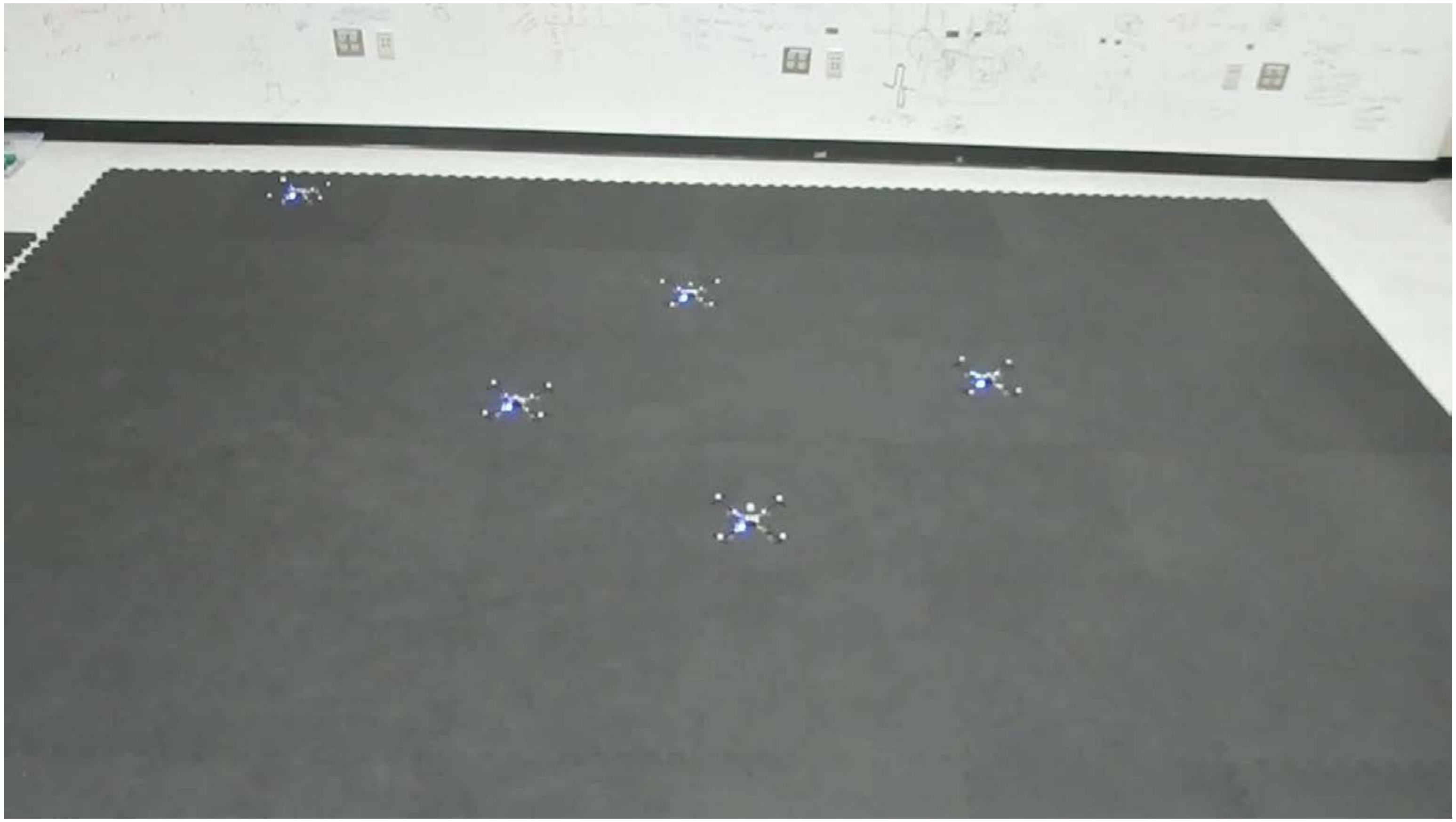


Ji, Azuma, Egerstedt, 2006. MacDonald, Egerstedt, 2011

Spatio-Temporal Formations

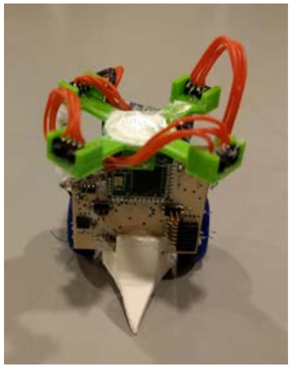
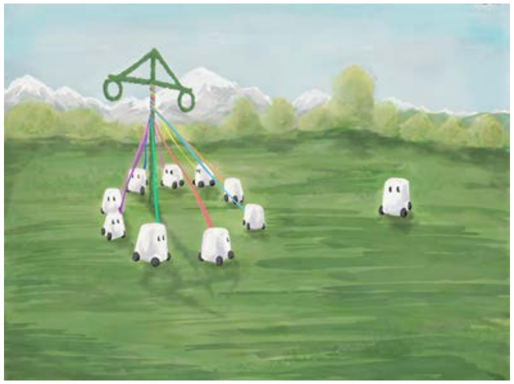
Chopra, Egerstedt, 2013.

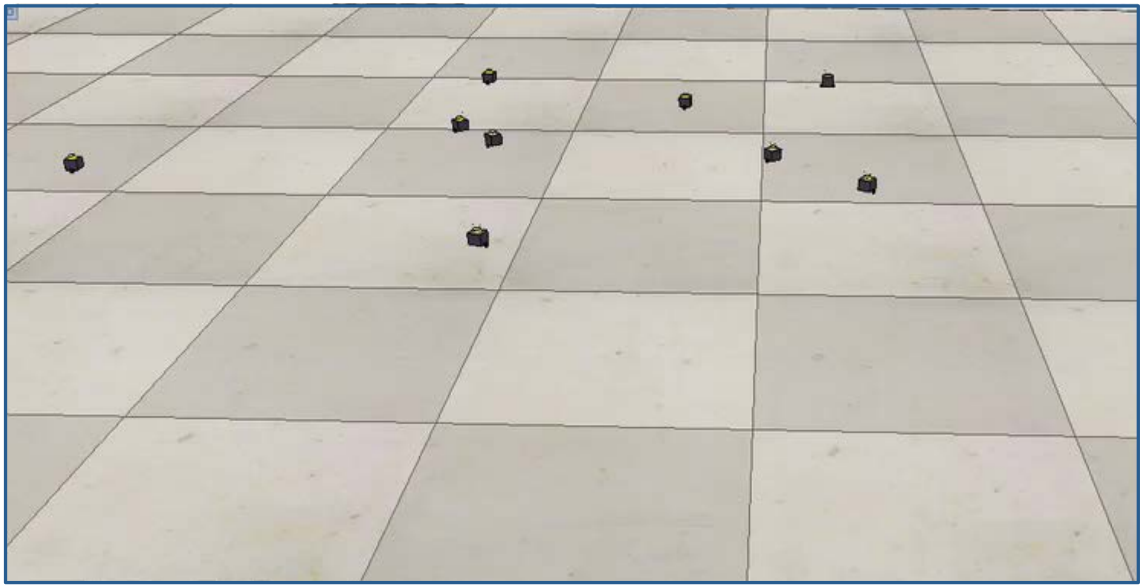
And In the Air…



Wang, Ames, Egerstedt, 2016

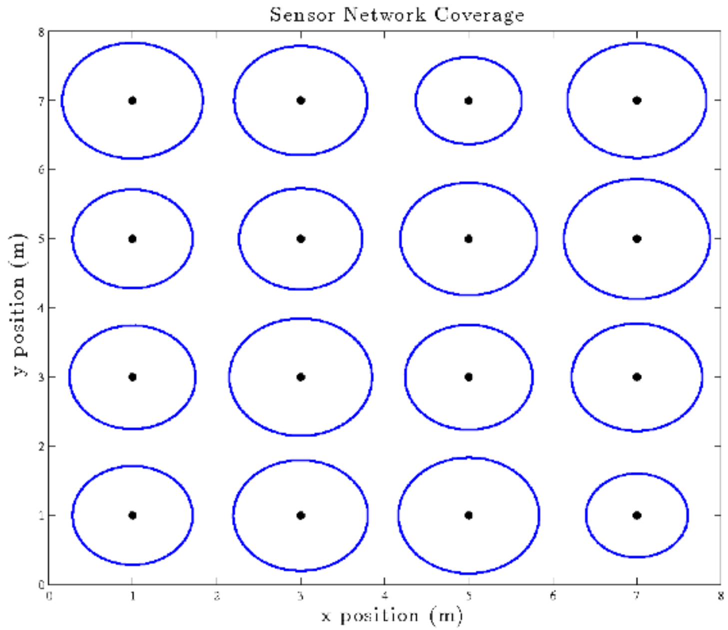
Coming to a Toy Store Near You…

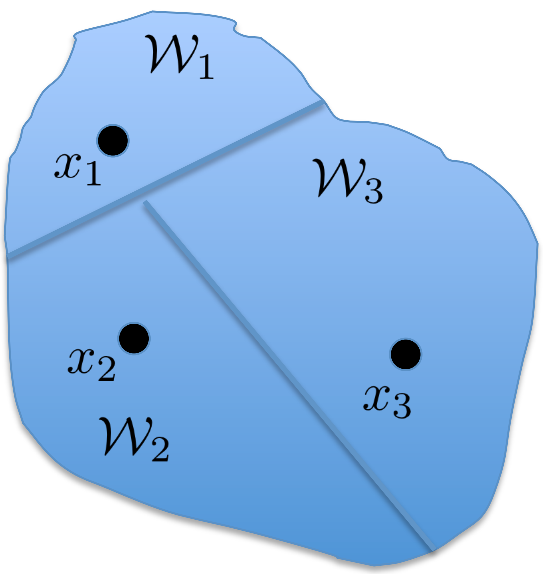






Formation Control v.3 ~ Coverage Control

- Objective: Deploy sensor nodes in a distributed manner such that an area of interest is covered





- Idea: Divide the responsibility between nodes into regions

Coverage Control

- The coverage cost:

$$J(x, \mathcal{W}) = \frac{1}{2} \sum_{i=1}^{N} \int_{\mathcal{W}_i} \|x_i - q\|^2 dq$$

- Simplify (not optimal):

$$\hat{J}(x) = \frac{1}{2} \sum_{i=1}^{N} \int_{\mathcal{V}_i(x)} \|x_i - q\|^2 dq$$

where the Voronoi regions are given by

$$\mathcal{V}_i(x) = \{q \in \mathcal{D} \mid \|x_i - q\| \leq \|x_j - q\|\}$$

Deployment

- Using a gradient descent (cost = weak Lyapunov function)

$$\dot{x}_i = -\frac{\partial \hat{J}}{\partial x_i} \;\Rightarrow\; \frac{d}{dt}\hat{J} = -\left\|\frac{\partial \hat{J}}{\partial x}\right\|^2$$

$$\dot{x}_i = -\int_{\mathcal{V}_i(x)} (x_i - q)dq = -\int_{\mathcal{V}_i(x)} dq\Big(x_i - \rho_i(x)\Big)$$

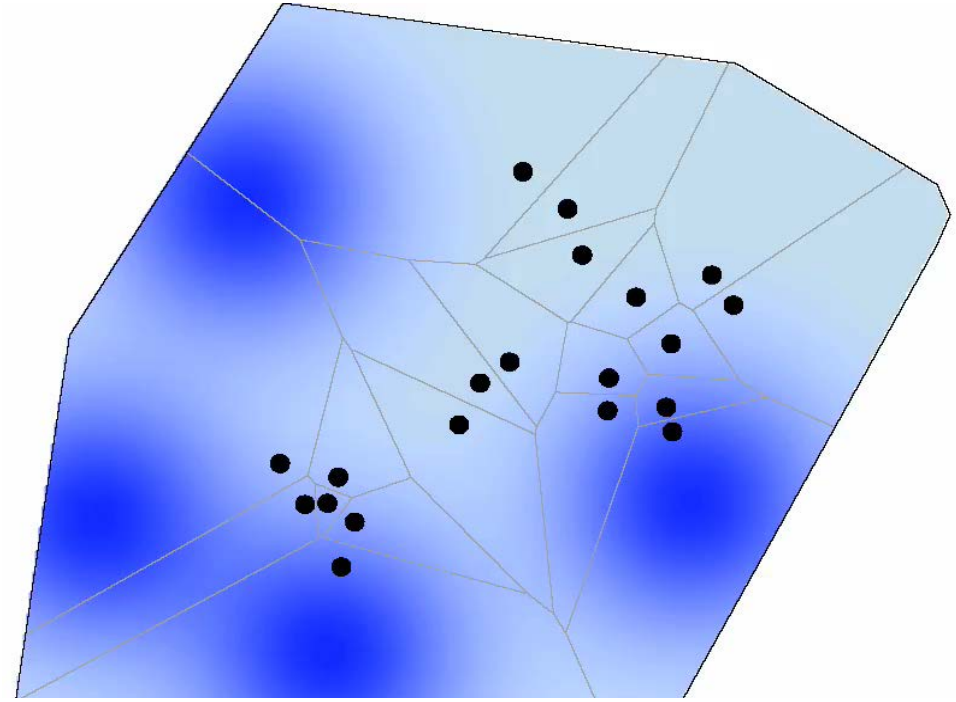
- We only care about directions so this can be re-written as Lloyd's Algorithm [1]

$$\dot{x}_i = \rho_i(x) - x_i$$

center of mass of Voronoi cell *i*

Deployment

- Lloyd's Algorithm:
  - Converges to a local minimum to the simplified cost
  - Converges to a Central Voronoi Tessellation



Courtesy of J. Cortes

Summary II

- Static Graphs:
  - Undirected: Average consensus iff G is connected
  - Directed: Consensus iff G contains a spanning, outbranching tree
  - Directed: Average consensus if consensus and G is balanced
- Switching Graphs:
  - Undirected: Average consensus if G is connected for all times
  - Directed: Consensus if G contains a spanning, outbranching tree for all times
  - Directed: Average consensus if consensus and G is balanced for all times
- Additional objectives is achieved by adding weights (edge-tension energies as weak Lyapunov functions)

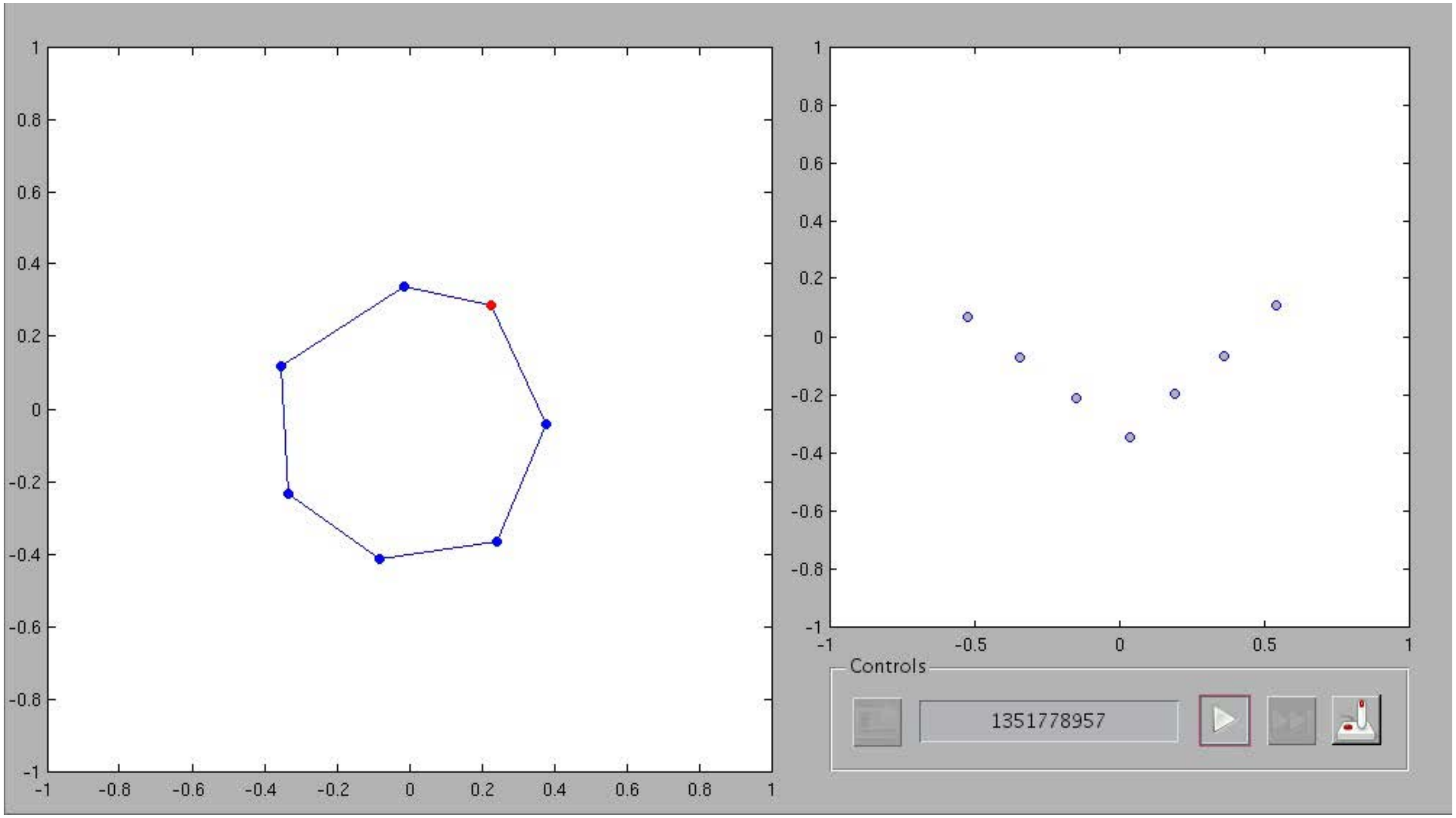
1. GRAPH-BASED ABSTRACTIONS
2. FORMATION CONTROL
3. INTERACTING WITH NETWORKS

Again: Why Swarming Robots?

- Strength in numbers
- Lots of (potential) applications
- Convergence of technology and algorithms
- ***Scientifically interesting!***

People will be part of the mix!

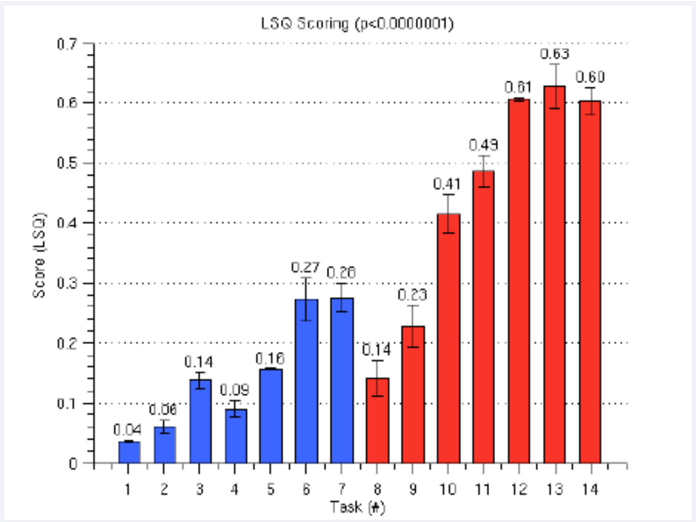
User Study



de la Croix, Egerstedt, 2014.

Results

- Performance “Error”, Difficulty, Workload



de la Croix, Egerstedt, 2014.

Results

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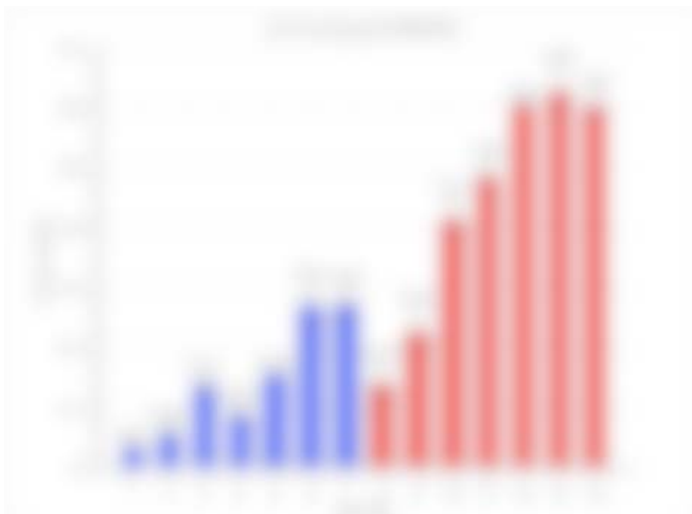


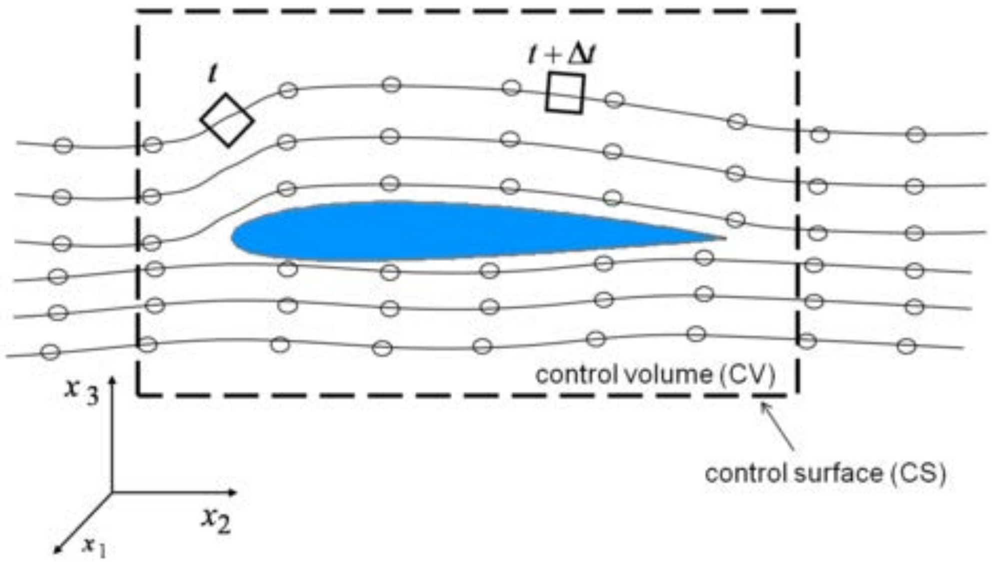
- **PEOPLE ARE REALLY BAD AT CONTROLLING SWARMS OF ROBOTS!**

de la Croix, Egerstedt, 2014.

A (Welsh) Mood Picture

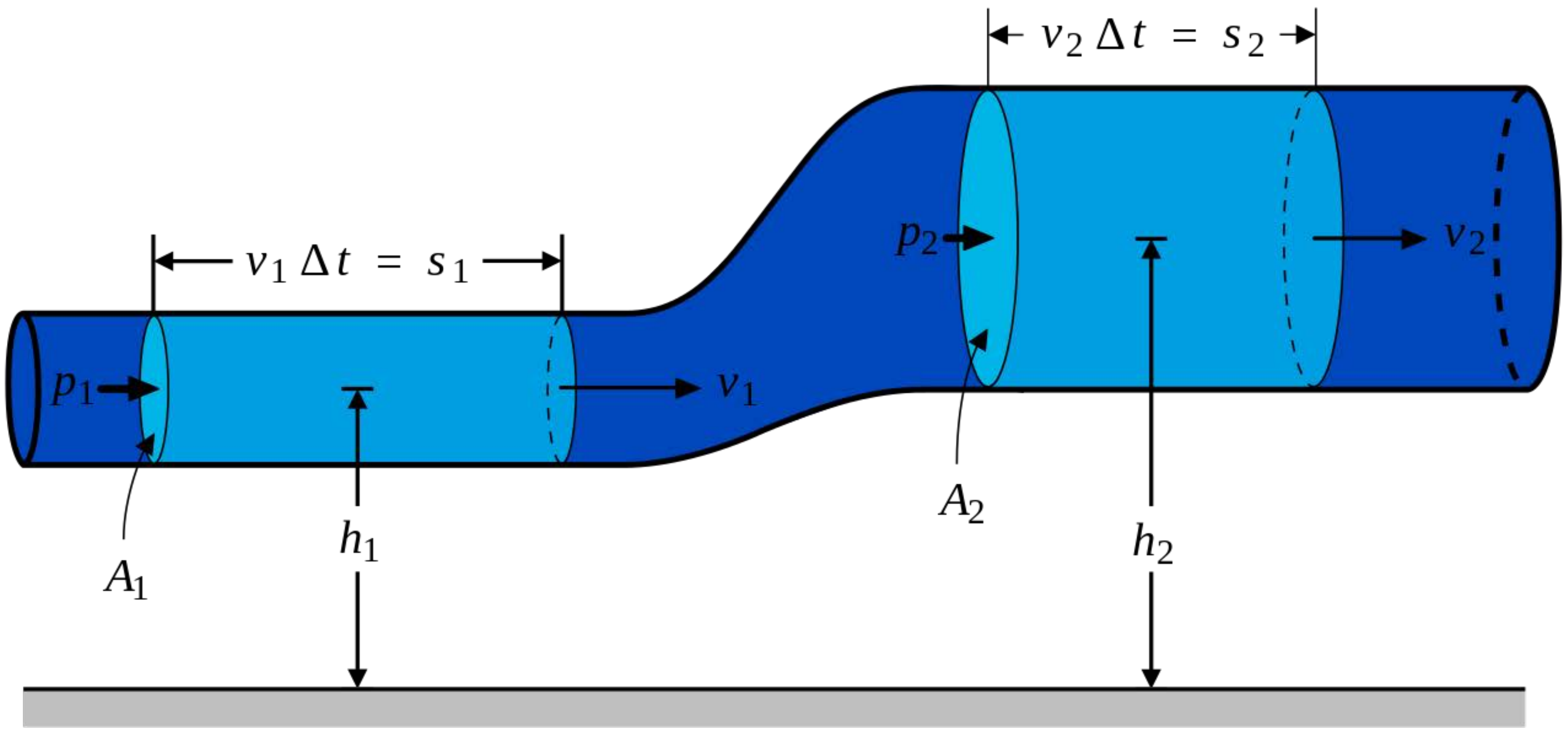
Duck Tales

Lagrangian Fluid Dynamics



Lagrangian Swarms:
- Formation Control
- Flocking, Rendezvous, and Swarming
- Coverage Control
- Boundary Protection and Containment
- ...

Eulerian Fluid Dynamics



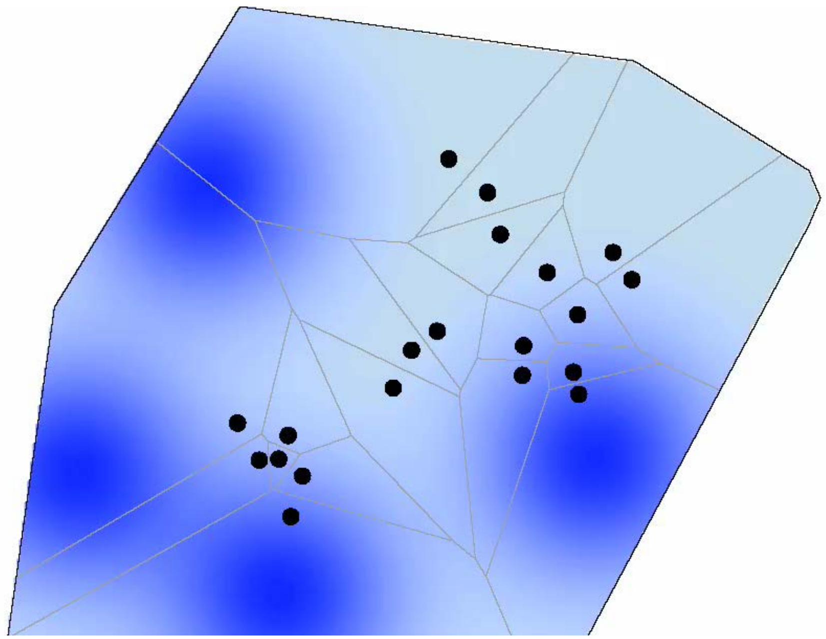
Eulerian Swarms?

Manipulating the Mission/Environment?

- Key idea: Human operator specifies areas of interest and the robots respond

$$J(x)=\sum_{i=1}^{N}\int_{\mathcal{V}_i(x)}\|x_i-q\|^2\phi(q)dq$$

specification



Gradient descent (Lloyd's algorithm)

$$\dot{x}_i=\rho_i(x)-x_i$$

center of mass of Voronoi cell i

Achieves a CVT:

$$x_i(t)-\rho_i(x(t))\rightarrow 0$$

Time-Varying Density Functions

- Need time-varying human inputs $\phi(q,t)$

$$\frac{d}{dt}\Big(x - \rho(x)\Big) = 0 \;\Rightarrow\; \dot{x} = \left(I - \frac{\partial \rho}{\partial x}\right)^{-1} \frac{\partial \rho}{\partial t}$$

- Problem 1: First need to get to a CVT
- Problem 2: Inverse not always defined
- Problem 3: Not distributed

$$\text{sparse}\left\{\frac{\partial \rho}{\partial x}\right\} = \text{sparse}\{G_{Delaunay}\}$$

$$\text{sparse}\left\{\left(I - \frac{\partial \rho}{\partial x}\right)^{-1}\right\} \neq \text{sparse}\{G_{Delaunay}\}$$

$$\left(\begin{array}{l} \bullet\ \text{Problem 4: Messy...} \\ \dfrac{\partial \rho_i^{(k)}}{\partial x_j^{(\ell)}} = \dfrac{\int_{\partial \mathcal{V}_{i,j}} \phi q^{(k)} \frac{x_j^{(\ell)} - q^{(\ell)}}{\|x_j - x_i\|} dq}{\int_{\mathcal{V}_i} \phi dq} - \dfrac{\int_{\partial \mathcal{V}_{i,j}} \phi \frac{x_j^{(\ell)} - q^{(\ell)}}{\|x_j - x_i\|} dq \int_{\mathcal{V}_i} \phi q^{(k)} dq}{\left(\int_{\mathcal{V}_i} \phi dq\right)^2} \end{array} \right)$$

Time-Varying Density Functions

$$\frac{d}{dt}\Big(x - \rho(x)\Big) = 0 \;\Rightarrow\; \dot{x} = \left(I - \frac{\partial \rho}{\partial x}\right)^{-1} \frac{\partial \rho}{\partial t}$$

- Problem 1: First need to get to a CVT
- Problem 2: Inverse not always defined
- Problem 3: Not distributed

$$\left(I - \frac{\partial \rho}{\partial x}\right)^{-1} = I + \frac{\partial \rho}{\partial x} + \left(\frac{\partial \rho}{\partial x}\right)^2 + \cdots$$

- Solution: Add a Lloyd term and use a truncated Neumann Series:

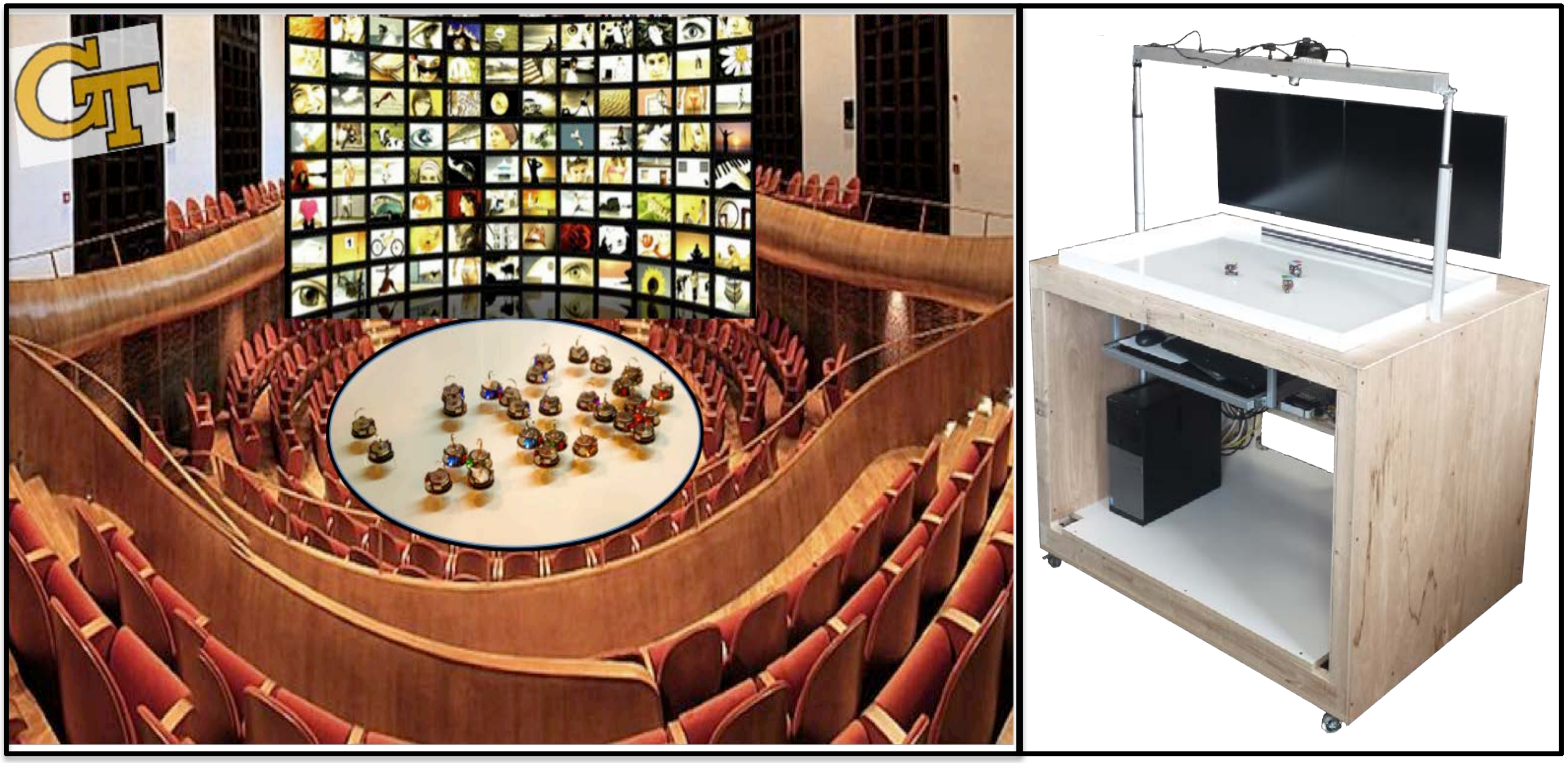
$$\boxed{\begin{gathered}\dot{x} = \underline{\left(I + \frac{\partial \rho}{\partial x}\right)} \left(\frac{\partial \rho}{\partial t} + \underline{\kappa(\rho - x)}\right) \\ x_i(t) - \rho_i(x(t)) \to 0^*\end{gathered}}$$

Lee, Diaz-Mercado, Egerstedt, TRO, 2015

Example 1: Precision Agriculture

Li, Diaz-Mercado, Egerstedt, 2015

Example 2: The Robotarium



MRI: A Shared, Remote-Access Multi-Robot Laboratory

So Far… [www.robotarium.org]

UT DALLAS

Distributed Formation Control

K. Fathian, N. Gans, M. Spong

Fault-Tolerant Rendezvous

H. Park, S. Hutchinson

東京工業大学
Tokyo Institute of Technology

Attitude Synchronization

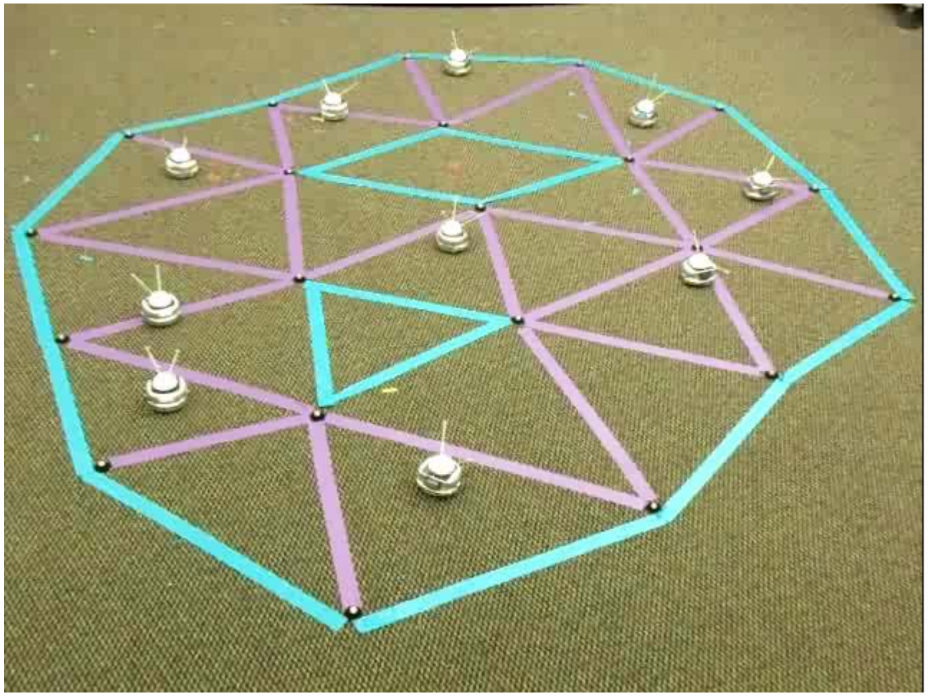
J. Yamauchi, M. Fujita

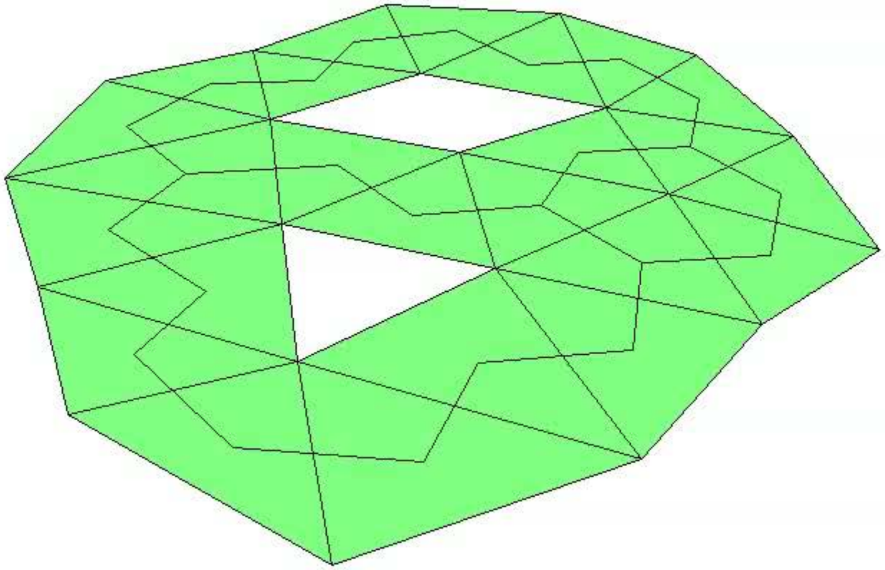
Since Jan. 2016:
115 robots, **21** research groups, **105** student projects

Example 2: The Robotarium

Example 3: Mind Control

Eulerian Approached Beyond Density Functions



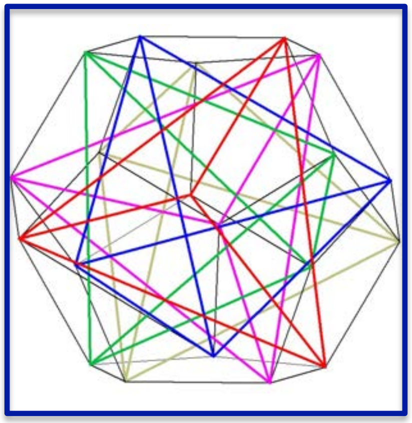


Kingston, Egerstedt, 2011

Summary III

- Lagrangian swarms at the level of the individual agents
- Eulerian swarms from the users' perspective:
  - Engage at the level of the team, not at the level of individuals
  - (For small team sizes, leader-follower control still works ok)
- Embedded humans (human-swarm interactions) is still a major area of research!

To Summarize

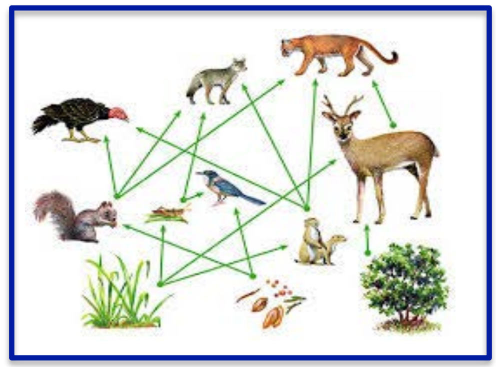


formations human-swarm interactions

Open issues:

complex dynamics?

malicious behaviors?



beyond geometry?

Thank You!

[1] S. Martinez, J. Cortes, and F. Bullo. Motion coordination with distributed information. *IEEE Control Systems Magazine*, 27 (4): 75-88, 2007.

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[4] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48 (6): 988–1001, 2003.

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[6] J.M. McNew, E. Klavins, and M. Egerstedt. Solving Coverage Problems with Embedded Graph Grammars. *Hybrid Systems: Computation and Control*, Springer-Verlag, pp. 413-427, Pisa, Italy April 2007.

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[9] P. Dayawansa and C. F. Martin. A converse Lyapunov theorem for a class of dynamical systems which undergo switching, *IEEE Transactions on Automatic Control*, 44 (4): 751–760, 1999.

Thank You!

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Collaborators:


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